Complete analysis of the BEAVRS benchmark using the GPU-based direct whole core calculation code CRANE*

Jun Li,[1] Guo-Hua Chen,[2,3] Qian Zhang,[1] Ruo-Han Zheng,[4] Li-Qin Zhang,[4] and Shao-Hong Zhang[1,†]

[1]*School of Physics, Zhejiang University, Hangzhou, 310027, China*
[2]*Shanghai NuStar Nuclear Power Technology Co. Ltd, Shanghai, 201101, China*
[3]*Advanced Technology Institute, Zhejiang University, Hangzhou, 310027, China*
[4]*Nuclear Power Institute of China, Chengdu, 610041, China*

The BEAVRS (Benchmark for Evaluation and Validation of Reactor Simulation) benchmark problem proposed by the Massachusetts Institute of Technology (MIT) computational reactor physics group is widely used by various institutions in the world to verify and validate their new generation direct whole core calculation codes. It enables analysts to develop an extremely detailed PWR core model, and carry out multi-physics coupled verification and validation for the reactor core analysis code. Although there are numerous publications in the literature reporting the validation results, few publishes a thorough validation results against all the measurement data of the problem. In this paper, the BEAVRS problem is solved using the CRANE code, which is a fully GPU-based deterministic direct whole core calculation code for PWR. It is demonstrated that direct whole core calculation with detailed core model can be performed on a mini server mounted with 10 consumer-grade RTX 3090 graphics cards, and the average time needed to complete neutronics and thermal hydraulics coupled analysis for single core state is just about one minute. Thorough verification and validation against all the measurement data shows that the predicted criticality, control rod bank worths, in-core detector signal distribution and the boron let-down curves of two cycles agree well with the measurements. These results indicate that even by exploiting the computing power of consumer-grade GPUs, direct whole core calculations with detailed core model for large commercial PWRs are now practically possible, and CRANE is ready for PWR practical applications in terms of both solution fidelity and speed performance.

Keywords: BEAVRS benchmark, direct whole core calculation, CRANE, GPU computation

I. INTRODUCTION

A nuclear reactor is a typical complex system with multi-physics coupling. Due to the high cost of conducting nuclear reactor experimental research, numerical simulation has always been an important means of investigating the behavior of nuclear reactors. With the rapid development of computer hardware performance, since the beginning of this century, researchers have made significant progress in conducting first-principle-based simulations of the multi-physics behaviors in nuclear reactors. A batch of new generation codes have been successfully developed, adopting either the deterministic approach or the probabilistic approach. For instance, code DeCART[1], nTRACER[2], MPACT[3], and NECP-X[4] are the deterministic ones primarily based on the method of characteristic (MOC) while code MC21[5], RMC [6] and OpenMC[7], JMCT[8] are the probabilistic ones employing the Monte Carlo (MC) method. All these codes are able to perform high-fidelity direct whole core calculations for PWRs without introducing any empirical hypothesis as in the conventional two-step analysis method.

BEAVRS [9] is a benchmark problem proposed by the MIT Computational Reactor Physics Group in 2013 to meet the needs of verifying and validating the new generation of reactor core analysis codes. This problem is based on a 4-loop pressurized water reactor designed by Westinghouse in the United States. It provides highly detailed as-is design data for the reactor and also measurement data for the initial two operating cycles to the public to allow for validation of methods developments. It enables analysts to develop an extremely detailed PWR core model, and carry out multi-physics coupled verification and validation for the reactor core analysis code. Since the previous light water reactor benchmarks can only be used to verify a single physical problem, and most of the institutions cannot have access to this kind of utility proprietary information, this problem is unique and valuable.

Considering the complexity of the problem and the computation cost that it would take to perform direct whole core multi-physics coupled calculation with very detailed core modeling, it is a challenging task to complete the high-fidelity analysis of the problem. So far, various institutions in the world have attempted to solve this problem using their new generation codes. However, the degree of completion for this problem varies among the institutions. Some have only completed the calculation for the zero power state at the beginning of the first cycle (BOC1), while others have completed both the BOC1 calculation and the first cycle core follow calculations. Only a few institutions have completed all the two cycles calculation provided in the benchmark problem. While for the validation of computational results against the measurement data, the degree of completion also varies among institutions. Most institutions have conducted comparisons of BOC1 critical boron concentrations and the boron letdown curve for the initial cycle, while only a few have carried out comparisons of control rod bank worth and in-core flux measurement results at different burnup of the reactor. So far, there is no literature published yet reporting the com-

* The work was granted by Nuclear Power Technology Innovation Center of China [Grant number: HDLCXZX-2022-HD-011].

† Corresponding author, Shao-Hong Zhang, School of Physics, Zhejiang University, Hangzhou, 310027, China. E-mail: zhangshh@zju.edu.cn

parison of control rod bank worth between the prediction and the measurement data obtained at the low-power physics test (LPPT) stage at the beginning of Cycle 2. Moreover, since the target computing platform for most of today's direct whole core calculation codes are CPU-based dedicated supercomputers, it is impractical for them to exactly follow the reactor power history due to the large computational cost required, people have to adopt the simplified or approximated power history when using these codes to perform the core follow calculation.

This work focuses on using the fully GPU-based deterministic direct whole core calculation code CRANE[10] to analyze the BEAVRS benchmark problem for assessing its solution fidelity and speed performance for PWR practical applications. It will be demonstrated that just by exploiting the computing power of consumer-grade GPUs, direct whole core calculation with detailed core model and reactor core follow calculation adopting the exact reactor power operation history are now practical possible. Thorough verification and validation of CRANE prediction results against all the measurement data of the benchmark problem will also be performed, parameters such as the control rod bank worth at the beginning of the second cycle, which have not been reported in previous literature, will also be presented with comparative results in this work.

The rest of the paper is organized as follows. Section II outlines the methodology that CRANE adopted for PWR neutronics, thermal hydraulics and fuel rod performance coupled calculation. Section III gives the necessary details about the CRANE BEAVRS core modeling. Section IV presents the validation results against all the measurement data from the two operating cycles of the problem, and also the comparisons of prediction accuracy of CRANE with that of relevant codes in the world. Section V gives the introduction of the CPU-GPU heterogenous hardware adopted for this study and the superior speed performance obtained by exploiting the computing power of today's consumer-grade GPUs, while Section VI concludes the paper.

II. CRANE METHODOLOGY

CRANE is an advanced deterministic direct whole core calculation code for PWRs. It has been developed in Shanghai NuStar Nuclear Power Technology Co., Ltd for several years as its next generation core analysis code targeting for commercial PWR applications. The code is able to perform high-fidelity reactor neutronics and thermal hydraulics coupled analysis. The main methodology implemented in the code is outlined in this section.

The neutronics module of CRANE has extensively inherited many methods and good practice from NuStar's 2D lattice physics code ROBIN[11], such as the 69-group cross section data library processed from the ENDF/B-VII.1 evaluated nuclear data file, the resonance calculation method applicable to the irregular geometries that integrates the traditional equivalence theory and the enhanced neutron current method, the use of pre-calculated correction factor tables to account for the resonance interference effects among major resonant nuclei. To achieve a three-dimensional neutron transport solution for the entire reactor with a fine resolution down to the sub-pin level, the 2D/1D coupled iteration method is adopted. MOC (Method of Characteristics) method is employed to solve both the 2D planar problem and also the axial 1D problem. Iteration acceleration is achieved by successively solving the pin-level multi-group and few-group partial current-based 3D Coarse Mesh Finite Difference (pCMFD) problem. Implicit trapezoidal method and Chebyshev rational approximation (CRAM) are used to solve the depletion problem, and the strategy of linear reaction rate method (LR) and logarithmic linear reaction rate method (LLR) are used to deal with the coupling calculation of neutron transport and depletion for normal fuel pins and Gd-bearing fuel pins respectively.

The thermal-hydraulic module of CRANE includes a subchannel analysis model for calculating the temperature and density of the coolant, as well as a fuel rod analysis model for calculating the temperature of the fuel pellet. The subchannel analysis model is developed with reference to the CTF code[12], it gives the temperature and density of the coolant around the fuel rod by solving the mass, energy, and momentum conservation equations for the two-fluid, three-field (i.e. fluid film, fluid drops, and vapor) system within the pin-level sub-channel control volumes. While the fuel rod analysis model is developed by employing the fuel performance analysis model that the FuSPAC code[13] used. Fuel rod behaviors under irradiation conditions caused by burnup degradation, swelling, expansion, creep and stress-strain are accurately simulated and the evolution of intra-pellet temperature distribution with burnup is generated.

It is worthy emphasizing that unlike most deterministic direct whole core calculation codes that runs on CPU-based dedicated supercomputers, CRANE by design is a fully-GPU-based code and its target computing platform is chosen as industry-affordable computer server mounted with 10 consumer-grade GPUs. As depicted below in Fig. 1 the flow chart of CRANE, where modules marked in green runs on GPU and modules marked in blue runs on CPU, it can be concluded that for CRANE almost all the computation-related modules run on GPU, only these modules that are performed only for once, such as input parsing, cross-section data library reading, geometric processing, and result editing, run on CPU. Moreover, in order to fully exploit the GPU computing power, the algorithms related to neutronics and thermal-hydraulics have been carefully designed for the GPU architecture, and extensive performance optimizations have been performed to ensure high parallel efficiency and the efficient use of limited GPU memories. For instance, to fully exploit the GPU computing power, long characteristic ray tracing rather than the cyclic characteristic ray tracing is adopted in CRANE and the 1D thermal conduction calculation for each axial node of all the fuel rods are solved in batches on GPU, with one GPU thread handling the computation of one node.

Moreover, in order to achieve high efficiency in multi-physics coupling iterations, CRANE utilizes direct shared memory for data exchange and adopts a unified mesh system for modeling, which makes the data exchange between

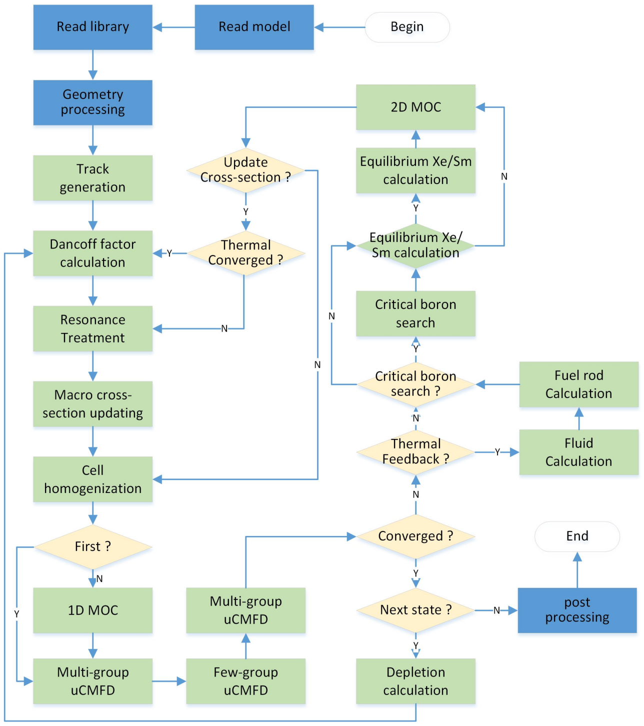


Fig. 1. Flow chart of CRANE

different modules extremely fast and eliminates the need to perform mesh and data mapping between different modules. This kind of highly-integrated coupling between neutronics and thermal-hydraulic modules further guarantee the exceptional computational speed performance of CRANE.

III. CORE MODELING

The reactor core specified in the BEAVRS benchmark problem is a four-loop Westinghouse PWR loaded with 193 optimized fuel assemblies of 17×17 lattice for the rated power of 3411 MWth. The benchmark specification provides all the detailed geometric dimensions and the material compositions for the major core constituents including fuel assemblies with different fuel enrichment, borosilicate glass burnable absorber, control rods, core baffle and barrel.

As the first step of the benchmark calculation, a virtual core model that faithfully represents the practical core is set up. Since CRANE employs Constructive Solid Geometry (CSG) method for modeling in the x-y plane, it is capable of faithfully modeling all the geometrical details of the problem without introducing any approximations. Fig. 2 and Fig. 3 show the core model that CRANE outputted by exploiting the Python visualization tool package.

More detailed information about CRANE BEAVRS core modeling is given as follows:

- In the radial direction, the 1/4 symmetric core is modeled, with the outer boundary setting at one fuel assembly width outside the active core; while in the axial direction, the code automatically generates the axial meshes according to the material heterogeneity with the maximum height of an axial layer setting to be 20 cm. In the end, the whole height of the problem is divided into 38 layers, of which 29 layers are in the active core. There are 9 spacer grids in total along the height, each height containing the spacing grid is set as a separate layer and all the spacer grid strap in these layers are explicitly modeled.

- In terms of spatial mesh division for the neutronics calculation, Fig. 4(a) and (b) show the flat source approximation mesh division schemes that CRANE adopted for a normal fuel pin cell and a burnable absorber pin cell respectively.

- The ray tracing parameters used for MOC calculation are as follows: for the radial 2D MOC calculation, the Tabuchi-Yamamoto optimum polar angle quadrature set[14] is employed, using 3 polar angles and 12 azimuthal angles for the octant of the solid angle sphere, and 0.05 cm ray spacing; while for the axial 1D MOC calculation, the Legendre quadrature set is employed, using 8 discrete angles in the range of 0-π and 0.5 cm mesh height for flat source approximation.

- Pin-wise thermal hydraulics (T/H) feedback is considered by solving the corresponding model for coolant and fuel rod, however, only the simple closed channel T/H module was used instead of the more time consuming elaborated sub-channel module since the cross flow between adjacent pin cells is small and has a negligible effect on core reactivity and power distributions.

- Convergence criteria: The criterion for critical boron concentration search is 0.5 ppm, the convergence criterion for calculating control rod worth and temperature coefficient is 0.2 pcm for k-effective, the convergence criterion for fission source is 5×10^{-4}, and the convergence criteria for T/H feedback calculations are 0.2 K for coolant temperature and 1 K for fuel pellet temperature respectively.

IV. VALIDATION RESULTS

1. HZP core calculation of Cycle 1

Table 1 shows the critical boron concentrations calculated by CRANE under 5 different control rod insertion states at the beginning of the Cycle 1 (BOC1) and the corresponding deviation from the measurements. The maximum deviation is 25 ppm, which meets well the requirement of American Nuclear Society (ANS) standards[15], i.e., the deviations between measured value and predicted value should be less than ±50 ppm. Reference[16] gives the standard deviation of the measured critical boron concentration, which is 22 ppm for all the 5 measurements. It can be seen that except for the

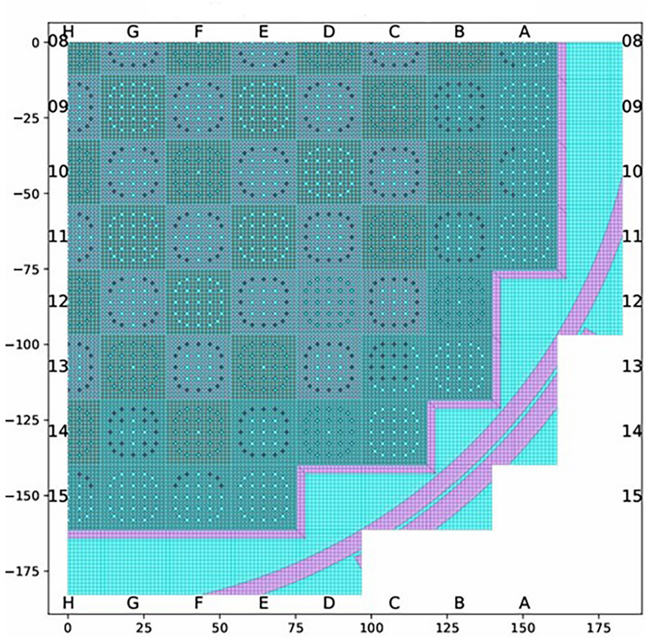


Fig. 2. Radial modeling of BEAVRS core

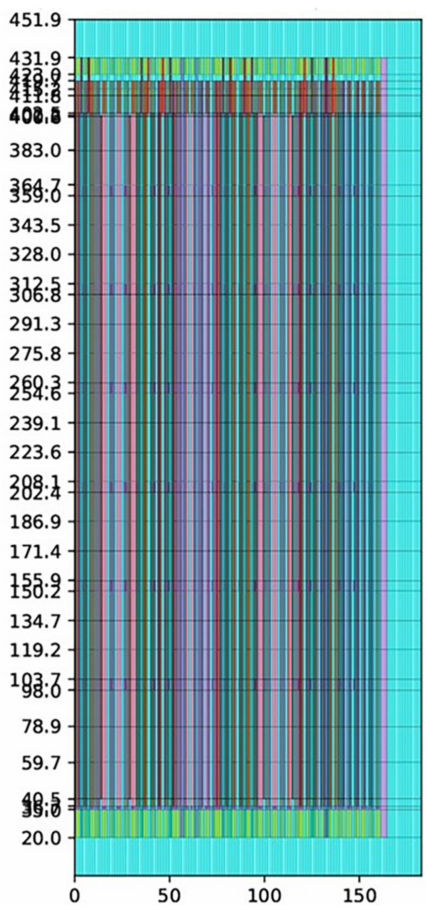


Fig. 3. Axial modeling of BEAVRS core

D-in state, where the deviation slightly exceeds the standard deviation of the measurements, all the other 4 deviations lie well within the range of the standard deviation of the measurement.

Moreover, since the variation in critical boron concentration under different control rod insertion states directly reflects the control rod worth. From the results in this table, it can also be inferred that all prediction deviations of control rod worth characterized by boron concentration variations fall within the range of standard deviation of the boron concentration measurements.

To compare the prediction accuracy of CRANE with that of relevant codes in the world, Table 1 also presents the prediction deviation of the deterministic code VERA[17] and Monte Carlo code RMC[18].The accuracy of CRANE for criticality prediction is fully comparable to that of VERA and RMC.

Table 2 shows the validation results of CRANE control rod worth prediction. The maximum relative deviation for the worth of individual bank is -10%, and the deviation for the total integral of predicted worths is -1.5%. The CRANE prediction accuracy meets the requirements of the ANS standards[15], i.e. the deviation for the worth of individual bank should be within $\pm 15\%$ or 100 pcm, whichever is greater, and the deviation for the total integral of control rod worths should be within $\pm 10\%$ (for Dynamic Rod Worth Measurement, the total worth should be within 8%).

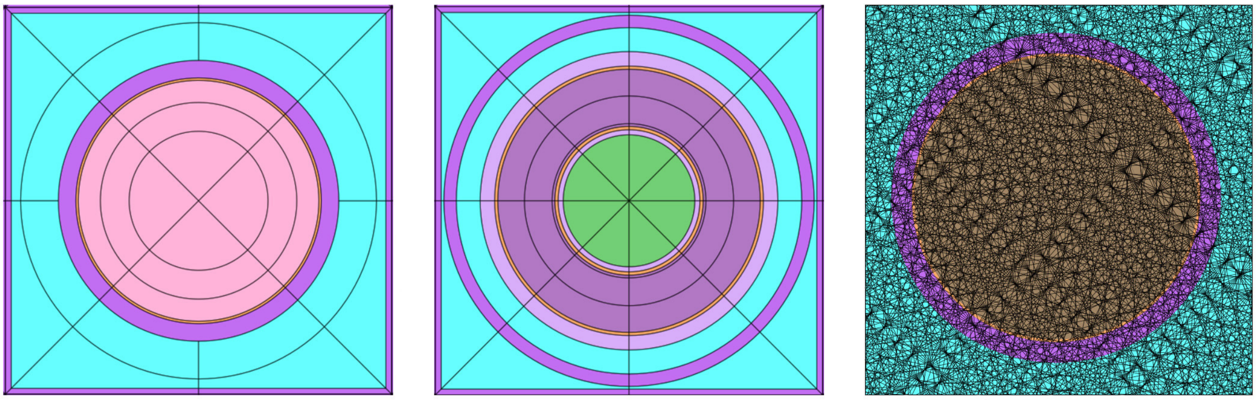
Similar to the comparison of the critical boron concentration, Table 2 also presents the prediction deviation of code VERA and RMC. In general, the prediction accuracy of CRANE is comparable to that of VERA and RMC. Moreover, when comparing the prediction deviation of each rod bank, it can be noticed that in all the three groups of deviations, the deviations of rod bank A and SE are relatively or exceptionally larger. It is believed that this is mainly caused by the relatively larger errors in the measured values for these two rod banks.

Table 3 shows the validation results for isothermal temperature coefficient (ITC). The benchmark problem gives 3 measured values at different control rod insertion states. For each core state, the CRANE prediction meets well the ANS standard requirements, i.e. the deviation should be less than ± 2 pcm/$^{\circ}F$.

2. *Core follow calculation of Cycle 1*

Fig. 5 gives the detailed power operation history of the initial cycle of BEAVRS problem, where the blue dots denote the time that the in-core neutron flux measurements is carried out. There are in total 24 in-core measurements performed during the whole cycle. For each in-core measurement, the benchmark problem provides detailed core state information, as well as the movable detector signals obtained from the fifty-eight instrumented assemblies. In addition, the benchmark problem also provides the boron letdown data at 29 different equivalent full power days (EFPD).

In order to validate the core depletion characteristics against the measured data, the core follow calculations were



(a) (b) (c)

Fig. 4. (a) Fine mesh of fuel pin. (b) Fine mesh of burnable poison rod. (c) 2D-MOC track.

Table 1. Comparisons of BOC1 HZP critical boron concentrations

Case	CBC[ppm]	Std. Dev.[ppm]	CRANE[ppm]	ΔCRANE[ppm]	ΔVERA[ppm]	ΔRMC[ppm]
ARO	975	22	988	13	1	-14
D in	902	22	927	25	13	-4
C, D in	810	22	833	23	4	-9
A, B, C, D in	686	22	698	12	-5	-25
A, B, C, D, SE, SD, SC in	508	22	513	5	-16	-39

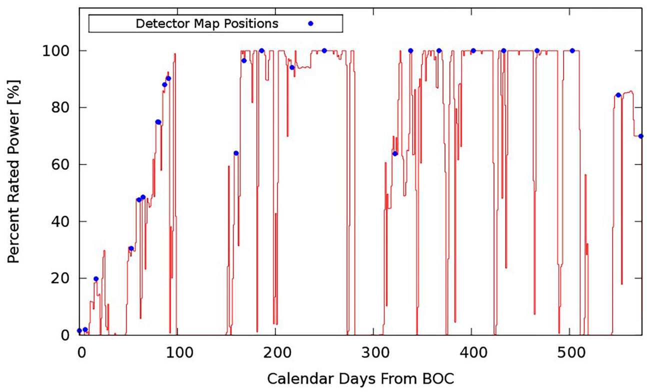


Fig. 5. Power history of Cycle 1

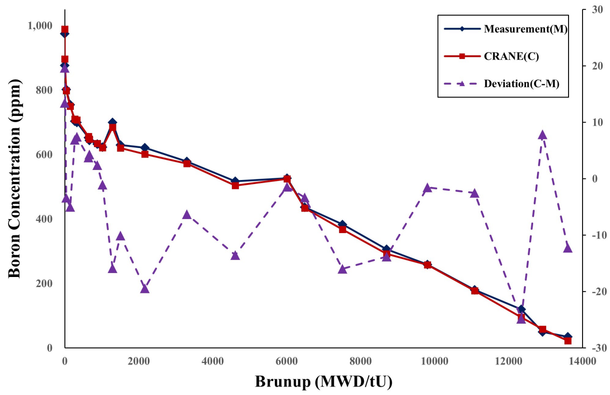


Fig. 6. Comparison of core follow critical boron concentration with measured data for Cycle 1

performed for the initial cycle of the BEAVRS core. The uniqueness of this work is that it adopts the exact power history to perform core follow calculation, since today's GPU computing power enables CRANE to do so. While for most existing CPU-based high-fidelity codes, people usually have to adopt simplified or approximated power history to complete the BEAVRS Cycle 1 core follow calculation. Because the large computational cost required for exactly tracking the complex power history makes it impractical for these codes to do so.

Fig. 6 gives the comparison of critical boron concentrations obtained from the core follow calculation with these specified by the problem for 24 different burnup states, where the in-core flux measurement is carried out. The CRANE prediction agrees well with the measurement and the deviations throughout the entire cycle are generally less than ±25 ppm.

The benchmark problem provides the reference boron letdown data for Cycle 1. In order to perform the boron letdown curve comparison, one has to obtain the critical boron concentration under the standard boron condition, i.e. the concentration under the all-rod -out (ARO) and hot-full-power(HFP) state. Although with the previous core follow boron concentration results available, one may obtain this boron concen-

Table 2. Comparisons of BOC1 HZP Control Rod Worths

Bank	CRW [pcm]	Std. Dev. [pcm]	Error[%]	CRANE[pcm]	ΔCRANE[%]	ΔVERA[%]	ΔRMC[%]
D	788	29	3.6	775	-1.6	-1.1	1.3
C	1203	32	2.6	1175	-2.3	4.2	2.5
B	1171	31	2.7	1247	6.5	2.1	-2.0
A	548	26	4.8	493	-10.0	5.7	-9.5
SE	461	25	5.5	416	-9.8	5.9	-3.0
SD	772	28	3.7	762	-1.3	0.9	3.4
SC	1099	31	2.8	1082	-1.5	-0.1	3.5
Total	6042	-	-	5950	-1.5	2.2	0.7

Table 3. Comparisons of BOC1 HZP Isothermal Temperature Coefficients

Case	ITC[$pcm/^{\circ}F$]	Std. Dev. [$pcm/^{\circ}F$]	CRANE[$pcm/^{\circ}F$]	ΔCRANE[$pcm/^{\circ}F$]	ΔVERA[$pcm/^{\circ}F$]	ΔRMC[$pcm/^{\circ}F$]
ARO	-1.75	0.54	-2.64	-0.89	-0.81	-0.53
D in	-2.75	0.54	-3.96	-1.21	-1.26	-0.80
C, D in	-8.01	0.54	-8.57	-0.56	-0.79	-0.04

tration by introducing theoretical corrections to consider the power and control rod effects, in this study, more straightforward method is applied, i.e. directly perform ARO HFP depletion calculation for Cycle 1. Fig. 7 shows the obtained results with the reference ones where the non-physical minus boron concentration at the end of cycle in the predicted value is because of that the predicted cycle length is slightly shorter than the reference one. One may notice that for the whole core lifetime of Cycle 1, the deviation of standard boron concentration lies within the range of ±25ppm, which is consistent with the above-mentioned core follow calculation results. As a supplement, Fig. 7 also shows the boron letdown curve predicted by the nTRACER[19] code, the results of CRANE and nTRACER are also in good agreement. Both CRANE and nTRACER slightly under-estimate the cycle-length of Cycle 1 and the situation of CRANE is slight better than that of nTRACER.

Reference [16] investigated the uncertainty of BEAVRS boron concentration measurement by analyzing the results of multiple measurements of boron concentration in the same day and comparing the deviation between the measured value and the calculation result of the analysis code currently used in engineering, and the conclusion was that the uncertainty is ±25 ppm, which is entirely consistent with the deviations that one may observe in Fig. 6 and Fig. 7. Therefore, it is concluded that CRANE is able to accurately predict the BEAVRS core reactivity variation for the entire lifetime of Cycle 1.

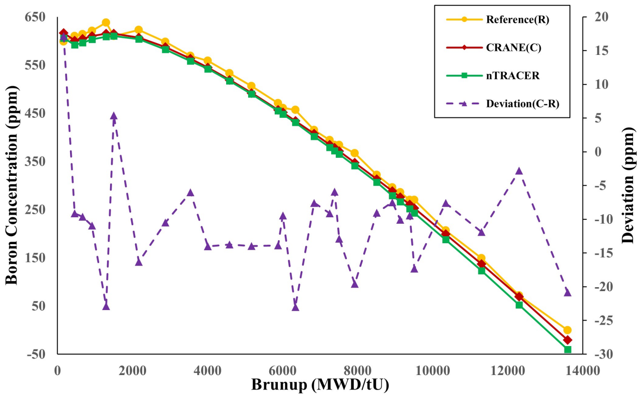


Fig. 7. Comparison of boron letdown curves in the Cycle 1 standard boron state

3. *In-core detector signal calculation of Cycle 1*

In this subsection, the accuracy of CRANE in calculating the 3D neutron flux distribution within the reactor core is validated against the in-core flux measurements provided by the BEAVRS benchmark problem. Since what the benchmark problem provided is just the normalized 3D detector response, not the detector current signal itself, the in-core detector signal is simulated in this study by putting a trace amount of U-235 in the instrumentation thimble to generate the fission rates at all the detector locations. The resulting detector fission rates are then normalized to generate a 3D distribution that can be compared with the measured 3D detector response.

As shown in Fig. 5, many in-core flux measurements of Cycle 1 were carried out during the power ascension process. The change of power level will cause the delayed change of xenon spatial distribution in the reactor, which will lead to the change of neutron flux distribution. Such transient fluctuations cannot be accurately calculated according to the operation history in days given by the benchmark. Therefore, just like the practice adopted in reference[17], only 14 sets of measurement data obtained at relative high-power level and

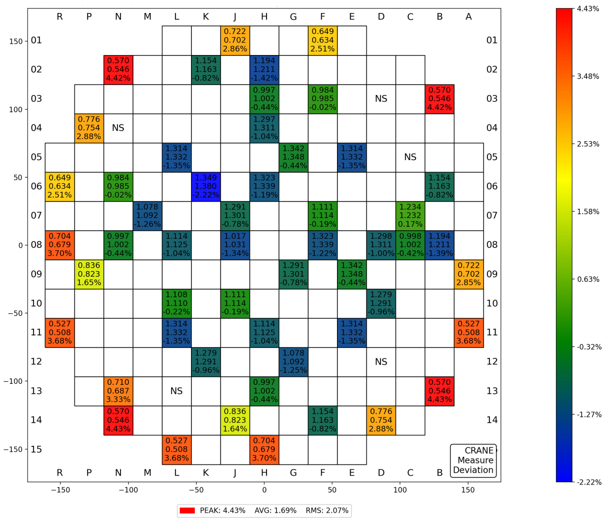


Fig. 8. Cycle 1 1295.50 MWdtU 64%FP detector signals relative difference

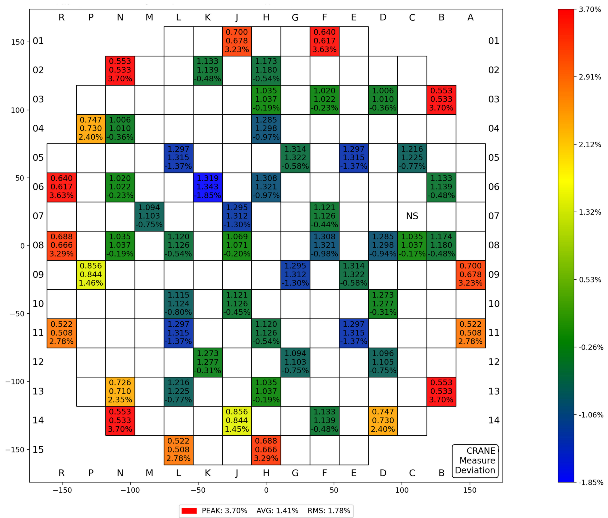


Fig. 9. Cycle 1 4613.50 MWdtU 100%FP detector signals relative difference

have been relatively stable are selected to be compared with. The comparison results are summarized in Table 4, where 2D RMS means the root mean square deviation between the predicted and measured 2D detector signal distributions. The 2D predicted and measured distributions are obtained by integrating the corresponding 3D detector signal distributions in the axial direction. The comparison given in Table 4 indicates that the accuracy of in-core detector signal prediction at different burnup for both CRANE and VERA are basically comparable.

To learn more detailed about the deviations in the detector signal distribution, Fig. 8- 10 each present a two-dimensional detector signal distribution deviation at a specific burnup point for the beginning, middle, and end of the cycle. From these comparisons, it can be seen that in general the accuracy of CRANE prediction is satisfactory, with the maximum relative deviation less than 5%. However, it can also be observed that compared to the measured signals, the signals predicted by CRANE exhibit a slight in- and out-ward tilt and the tilt decreases as the fuel burnup feedback increases.

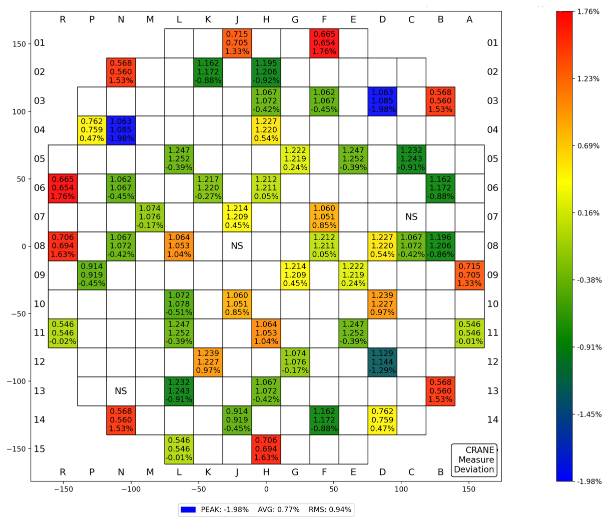


Fig. 10. Cycle 1 12341.20 MWdtU 100%FP detector signals relative difference

4. HZP core calculation of Cycle 2

At the beginning of Cycle 1, all the fuel loaded into the core are fresh fuels. For Cycle 2, which is a reloaded core, there are both depleted fuels and also fresh fuels loaded into the core at the beginning of the cycle. Engineering functions such as deplete fuel shuffling and removing the borosilicate glass burnable absorber rod from the deplete fuel from Cycle 1 are necessary for a code to perform Cycle 2 calculations. Therefore, the Cycle 2 problem provides not only a case to further validate the core depletion functionality of a core analysis code but also a good benchmark to check whether the code possesses the necessary functionality such as fuel shuffling for engineering applications.

The CRANE code was adopted to perform HZP core calculation of Cycle 2 of BEAVRS problem. Critical boron concentration, isothermal temperature coefficient, control rod worth corresponding to the measurement were predicted and the comparison results are summarized in Table 5 and Table 6 respectively. It can be seen that the predicted critical boron concentration and temperature coefficient are in good agreement with the measurement, and the degree of deviation between the prediction and measurement is similar to that of the first cycle. Meanwhile, it can also be seen that CRANE and VERA have comparable prediction accuracy for criticality calculation.

When looking at the validation results of CRANE control rod worth prediction, it is noticed that in general the prediction accuracy is similar to that of the first cycle and meets the

Table 4. Comparison of 2D detector signal distributions at different burnup of Cycle 1

Depletion [MWD/tU]	Power [%]	CRANE 2D RMS [%]	VERA 2D RMS [%]
1023	99	2.02	1.67
1296	64	2.07	3.42
1507	100	2.52	0.84
2163	100	2.73	1.18
3297	94	1.92	0.82
4614	100	1.78	0.90
6012	64	1.11	1.18
6490	100	1.90	1.23
7508	100	1.22	0.88
8701	100	1.12	0.99
9803	100	1.15	3.24
11084	100	1.06	1.12
12343	100	0.94	1..21
12915	84	1.38	1.48
Cycle average	-	1.75	1.46

requirements of the ANS standard. However, unlike the first cycle, there are more banks of rod with deviations reaching around 10%, and the maximum relative deviation has reached 12.7%. The reason for this phenomenon, as believed in this study, is mainly due to the errors in the measurements themselves. Looking at the standard deviation errors of measurement given in Table 6 and Table 2, it can be found that the error is further increased at Cycle 2, especially for banks with relative small control rod worth, such as bank SD, SC and SA, where the measurement error itself exceeds 7%.

As mentioned earlier there is no existing literature published the control rod worth prediction for Cycle 2 of BEAVRS problem, therefore there is no validation results of other code provided in Table 6.

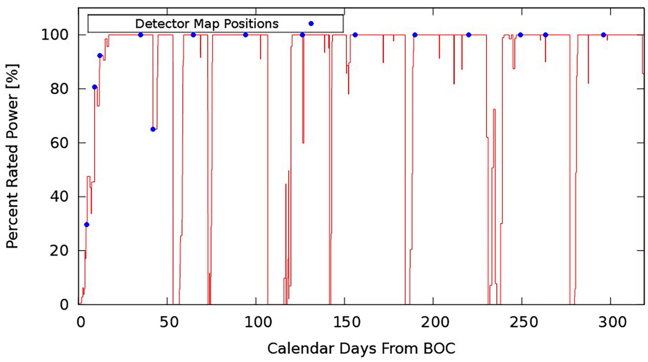


Fig. 11. Power history of Cycle 2

5. *Core follow calculation of Cycle 2*

As the practice of the Cycle 1, Cycle 2 core follow calculation adopting the exact power history depicted in Fig 11 was performed, and the CRANE predicted boron concentrations were compared against the measurements specified for 14 burnup states where the in-core flux measurement is carried out. Moreover, ARO HFP core depletion calculations were also performed and the obtained boron concentrations were compared against the reference boron letdown data as well as the ones predicted by the nTRACER code. Comparison results are shown in Fig 12 and Fig 13 respectively. It can be seen that except for one isolated instance of core follow calculation where the deviation of boron concentration exceeds 35ppm, all the other deviations between the CRANE-predicted and the measured critical boron concentration for both core follow calculation and boron letdown curve calculation are less than 25 ppm throughout the entire cycle. Considering that the uncertainty of BEAVRS boron concentration measurement is $\pm$25 ppm, once again, one may conclude that CRANE is able to predict the BEAVRS core reactivity variation for the reload cycle with satisfying accuracy. The comparable accuracy between CRANE and nTRACER results is also demonstrated.

6. *In-core detector signal calculation of Cycle 2*

For Cycle 2, there are in total 14 in-core flux measurements performed during the whole cycle, and all these measurements are simulated by CRANE. The obtained 2D in-core detector signal distributions are compared against the measurements and the results are summarized in Table 7. To facilitate the performance assessment between CRANE and other high-fidelity codes, the in-core detector signal validations results of the deterministic code VERA and the Monte Carlo and

Table 5. Comparisons of Cycle 2 Critical Boron Concentrations and Isothermal Temperature Coefficient

Test type	Case	Measurement	Std. Dev.	CRANE	△CRANE	△VERA
CBC [ppm]	ARO	1405	22	1401	-4	-8.6
CBC [ppm]	C in	1273	22	1300	27	21.4
ITC [pcm/$^{\circ}F$]	ARO	1.71	0.54	1.27	-0.44	-

Table 6. Comparisons of Cycle 2 Control Rod Worth

Bank	Measurement[pcm]	Std. Dev.[pcm]	Error[%]	CRANE [pcm]	△CRANE[%]
D	426	25	5.8	470	10.21
C	1014	30	3.0	1028	1.34
B	716	28	3.9	705	-1.60
A	420	25	5.9	408	-2.75
SE	438	25	5.7	445	1.56
SD	305	23	7.5	344	12.69
SC	307	23	7.5	336	9.44
SB	781	29	3.6	793	1.54
SA	326	23	7.1	356	9.06
Total	4733	-	-	4883	3.17

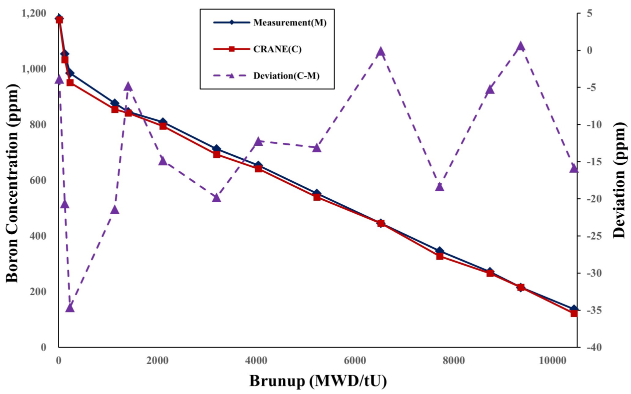


Fig. 12. Comparison of measured boron concentration and CRANE results in Cycle 2

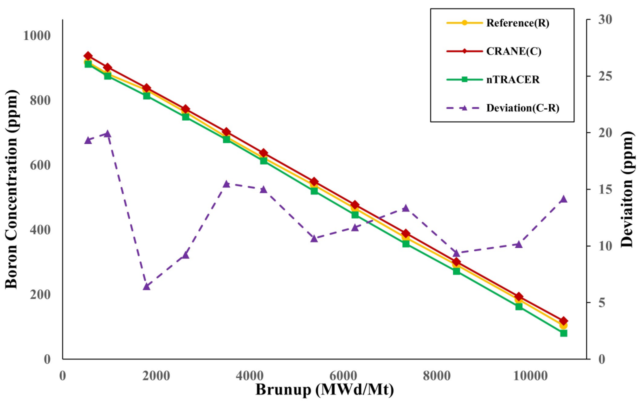


Fig. 13. Comparison of boron letdown curves in the Cycle 2 standard boron state

thermal hydraulics code coupled system (MCS+CTF)[20] are also provided in Table 7. Based on these comparison results, it can be concluded that CRANE is able to accurately predict the in-core flux distribution and its variation with fuel burnup for the whole cycle. The accuracy of CRANE to predict the in-core detector signal is fully comparable to that of VERA, and generally better than the MCS/CTF system. Nevertheless, unlike the VERA case where the RMS error at the first two burnup points are evidently larger, the error of CRANE is rather stable throughout the cycle.

As the practice of Cycle 1, three representative detector signal deviation distributions are shown in Fig. 14 to Fig. 16. Compared with the ones for Cycle 1, the results of Cycle 2 are overall better,the relative deviation of all the results is within 5% and the in- and out-ward tilt existed in the deviation distribution for Cycle 1 are no longer existed for this cycle.

V. SPEED PERFORMANCE AND POTENTIAL FOR PRACTICAL APPLICATIONS

As the core analysis method shifts from the conventional two-step procedure to the direct whole core calculation involving first-principle-based simulations of the multi-physics behaviors in nuclear reactors, the computation load increases significantly. It is estimated that the computation load of the new method is at least 2 to 3 orders of magnitude greater than that of the conventional method. Desktop personal computer or workstation, which are the computing platform for the con-

Table 7. Cycle 2 deviation of detector signals under different burnup

Depletion [MWD/tU]	Power [%]	CRANE 2D RMS [%]	VERA 2D RMS [%]	MCS+CTF 2D RMS [%]
12.92	29	1.67	3.11	4.08
125.45	80	1.22	3.22	2.43
224.65	100	1.42	1.57	2.90
1126.14	100	1.06	1.11	3.10
1395.39	65	1.48	-	2.99
2091.83	100	1.04	1.04	2.90
3169.63	100	1.04	1.15	2.70
4006.52	100	1.19	1.14	2.65
5178.51	100	1.14	1.12	1.47
6463.87	100	1.31	1.18	2.59
7645.03	100	1.40	1.38	3.03
8650.31	100	1.18	0.95	2.81
9272.15	100	1.17	1.16	3.11
10338.69	100	1.04	1.11	1.17
Cycle average	-	1.26	1.43	2.71

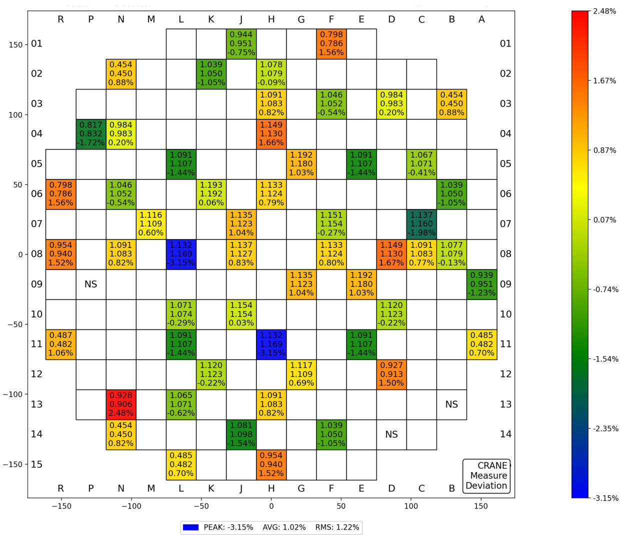


Fig. 14. Cycle 2 125.45 MWdtU 80%FP detector signals relative difference

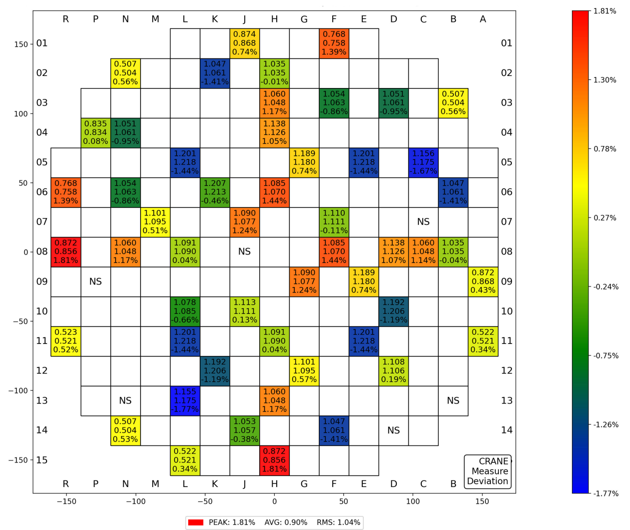


Fig. 15. Cycle 2 3169.63 MWdtU 100%FP detector signals relative difference

ventional method, are no longer suitable for the new method due to their limited computing power and memories. High-performance computing (HPC) clusters or supercomputers equipped with many CPUs are chosen by many institutions in the world as the computing platform for the new generation PWR core analysis codes. However, due to the complexity of the problem, even by exploiting the computing power of today's HPC clusters or supercomputers, it is still a challenging task to reduce the computing time of the direct whole core calculation code to a practical level such that it can be used in routine design analysis. For instance, to simulate the BEAVRS benchmark, VERA takes 50min (760 CPU-hrs) on average to complete the corresponding one state core analysis on a cluster with 880 2.3 GHz Intel Xeon processors with 4 GB RAM [17]; while nTRACER takes 3 hours for a single-step calculation running on a cluster with 28 computing nodes mounted with dual 2.67 GHz INTEL XEON X5650 processors yielding 12 cores per node[19].

Moreover, as can be deduced from the above-mentioned VERA and nTRACER results, in order to reduce the computing time of a new generation code to a practical level, supercomputers mounted with thousands of cores are necessary. Unfortunately, such large-scale HPC clusters are usually available only at national laboratories, not at nuclear design

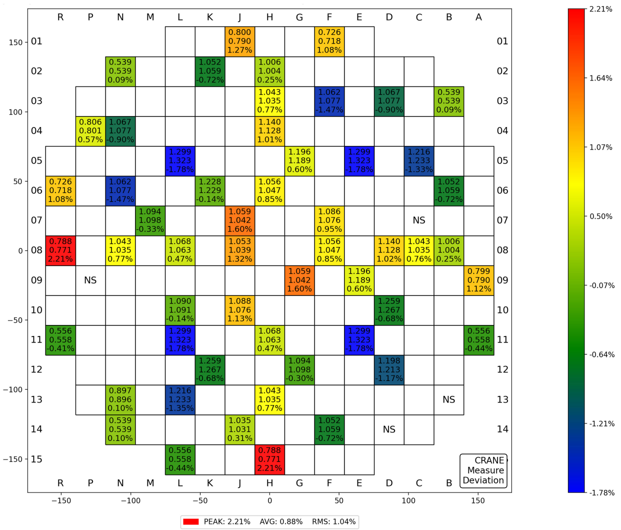


Fig. 16. Cycle 2 10338.69 MWdtU 100%FP detector signals relative difference

institutes, not to say ordinary research groups. Therefore, both the computing time and the platform availability issue pose a key obstacle that hinders the widespread use of these new generation codes based on HPC cluster or supercomputer based.

Meanwhile, in the past two decades, the computing power of GPUs has exhibited a much stronger growth than that of CPUs. The computing power that used to be provided by multiple CPU nodes can now be provided by a single GPU card. GPUs provide much higher computing power than CPUs for the same energy consumption. It thus provides the researchers a new platform to consider for the development of the direct whole core calculation codes for PWRs, i.e. adopting heterogeneous computing that utilizes more GPUs than CPUs. In this regard, the Reactor Physics Lab of Seoul National University has conducted valuable exploration[21][22]. It is demonstrated that after proper porting the legacy CPU code to GPU, significant speedup can be achieved for an industrially affordable GPU-based heterogeneous computing platform mounted with consumer-grade GPUs.

In this study, the BEAVRS benchmark problem is solved using the CRANE code. One of the uniqueness of CRANE is that it is by design a fully-GPU-based code. As depicted in the flow chart of CRANE (Fig 1), for CRANE, the GPU computing is not just for acceleration, but the main force of the computation, since almost all the computation-related modules run on GPUs. To resolve the above-mentioned issue of platform availability, the target computing platform of CRANE is chosen as industry-affordable computer server mounted with consumer-grade GPUs. All calculations for this work are completed on a small server equipped with 10 RTX 3090 graphics cards, and the peak power consumption of the server is 4kW. The statistics indicates that to complete all the validations for the parameters specified in BEAVRS benchmark, 738 different core states were analyzed and it took 15.7 hours in total. The average time needed to complete one state multi-physics coupled high-fidelity core analysis is 63.5s and 101.2s respectively for cycle 1 and cycle2.

This speed performance is significant because it fully demonstrates the power of GPU computing, which is that it can reduce the time required to complete a high-fidelity simulation of a large commercial pressurized water reactor to the minute level, without resorting to dedicated supercomputers, but merely by using low-cost consumer-grade graphics cards. A summary of the key computation conditions and corresponding speed performances employed by several direct whole core calculation codes for the BEAVRS benchmark problem is presented in Table 8. The data in the table reveals that the outstanding speed performance of CRANE is achieved by using more energy groups and more axial layers than other codes.

Another advantage of developing the new generation code based on the CPU-GPU heterogeneous platform is the ability to continuously leverage the rapidly advancing GPU technology to enhance the computational speed of the code. The RTX 3090 graphics card used in this paper for the BEAVRS problem computation was released in 2020, and since then, NVIDIA company has successively released the RTX 4090 and 5090 graphics cards with superior performance. The author ever had a chance to run the CRANE code on another platform equipped with RTX 4090 GPUs, and the new computation time is just about 50-60% of that for the current 3090 platform, i.e., the high-fidelity calculation of a single core state of a large commercial reactor, which currently takes 1-2 minutes to complete, can basically be finished within 1 minute on the new 4090 platform. Considering the rapid increase in the performance of GPU graphics cards, the author believes that more and more computation-extensive CPU-based legacy codes will be ported to the GPU-CPU heterogeneous plat form, and CRANE, as a fully GPU-based pioneer code, will have a good chance to be widely used as a routine tool in the nuclear power industry.

VI. CONCLUSION

The BEAVRS benchmark problem has been successfully analyzed with the CRANE code, which is a fully GPU-based deterministic direct whole core calculation code targeted for PWR practical applications. Detailed reactor core models are set up, which faithfully represent the practical core of the initial two cycles. Core follow calculations are performed by adopting the exact power history of the reactor. Validation against all the measurement data shows that the predicted criticality, control rod bank worths, in-core detector signal distribution and the boron let-down curves of two cycles agree well with the measurements. The deviations of the predicted results are all within the acceptable range for engineering applications. Moreover, it is demonstrated that by fully exploiting the GPU computing power, direct whole core calculation without resorting to the dedicated supercomputer mounted with thousands of cores is possible. Direct whole core calculation with detailed core model can be realized on an industry-affordable mini computer server mounted with 10

Table 8. Summary of calculation conditions and speed performance for different codes

Code	CRANE	VERA[17]	nTRACER[22]	nTRACER[22]
Platform	Intel Xeon Platinum 8375C CPU, 64 cores with 1 TB RAM, 10 RTX 3090	Intel Xeon 2.3 GHz CPU, 880 cores with 4 GB RAM	Intel Xeon E5-2640 v3 CPU, 288 cores with 128 GB RAM,	Intel Xeon E5-2630 v4 CPU, 180 cores with 256 GB RAM, 36 RTX 2080
# of Energy groups	69	51	47	47
# axial planes	38	34	36	36
Execution time per core state	1~2min	50min	2.1h	15min

consumer-grade RTX 3090 graphics cards, and the average time needed to complete multi-physics coupled high-fidelity analysis for single core state point is just about one minute. These results indicate that CRANE possesses high solution fidelity and superior speed performance, it has good prospects for PWR engineering applications.

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